Integration of Weighted Terminological Concepts and Vague Knowledge in Ontologies for Decision Making

Nadine Mueller[0000-0002-1658-5002] and Klemens Schnattinger[0000-0003-4565-7930]

Baden-Wuerttemberg Cooperative State University
{schnattinger,muellnad }@dhbw-loerrach.de

Abstract. Description logics (DLs) are a well-known family of logics for managing structured knowledge. They are the basis for widely used ontology languages. Experience with the use of DLs in applications has shown that their capabilities are not sufficient for every domain. In particular, the decision-making process requires the assessment of two different, sometimes even contradictory influences on decision factors. On the one hand, there are items that belong to certain classes or fulfill certain roles within logically complex constructs, but these memberships are to some extent vague. On the other hand, individual preferences can change depending on the person who drives the decision-making process. Therefore, the challenge when building a framework of decision making, is to take these influencing variables adequately into account by depicting and incorporating both aspects. The paper shows how these requirements can best been modelled by combining fuzzy description logic and weighted description logic. Whereas the first meets the requirement to represent vagueness and ambiguity in ontologies, the second is able to express individual preferences. In addition, the paper shows how to engineer an appropriate and suitable architecture for this purpose.

Keywords: Ontology Learning, Weighted Description Logic, Fuzzy Logic, Decision Making

1 Introduction

In many cases of decision making, expert knowledge is required. Human experts can identify structural patterns of decision situations in order to model decision processes [1]. From a cognitive-psychological point of view, decision making requires heuristics that ignores some of the information to make decisions more quickly, more economically or more accurately. Being able to work with vague information is critical when dealing with systems that are described by complex ontologies and consist of many instances [2]. Decision making and argumentation interact between processes that use logical thinking or heuristic reasoning. Therefore, it can be argued, that intuitive processes allow access to some form of logical reasoning. But it is also possible, that logic and rationality can be conceived as the domain of explicit higher-level forms of processing.

To formalize this knowledge, description logics offer a powerful tool to structure knowledge and support reasoning. When making decisions, it is often necessary not only to fulfill a set of equivalent requirements but also to take individual preferences into account. This requires an extension of common knowledge bases, the so-called decision bases, which are initially based on multi-attribute utility theory (MAUT) [3]. Since then, various approaches have emerged. Among others, the application of logic for decision and utility theoretical problems are given in [4-6]. In situations where ambiguity occurs, an acknowledged approach is to augment the framework by fuzzy logic, see [7,8]. However, in cases where individual preferences encounter vague knowledge and assertions, neither decision bases nor fuzzy description logic can satisfy these paradigms by its own. To close the gap, this paper offers a framework to model ambiguity and individual preferences at the same time. It brings together fuzzy description logic with weighted description logic. For the ease of understanding, we will first introduce the architecture which has been used in this specific context. Afterwards, to get a fine grasp of the combined framework of weighted description logic and fuzzy description logic, we will familiarize the reader with both separately. Initially, we establish the basics of weighted description logic. Subsequently, we present the fuzzy description logics and focus how it supports the modelling of ambiguous and vague knowledge. At the same time, the demarcation to probabilistic settings is highlighted. After combining these two approaches, our fuzzy decision base framework is introduced. Finally, we show how this framework can support the decision-making process within the respective architecture.

2 Preliminaries

The following sections present our architecture for opinion and consensus mining OMA, classical description logic and two extensions, the weighted description logic and fuzzy description logic.

2.1 Opinion & Consensus Mining Architecture OMA

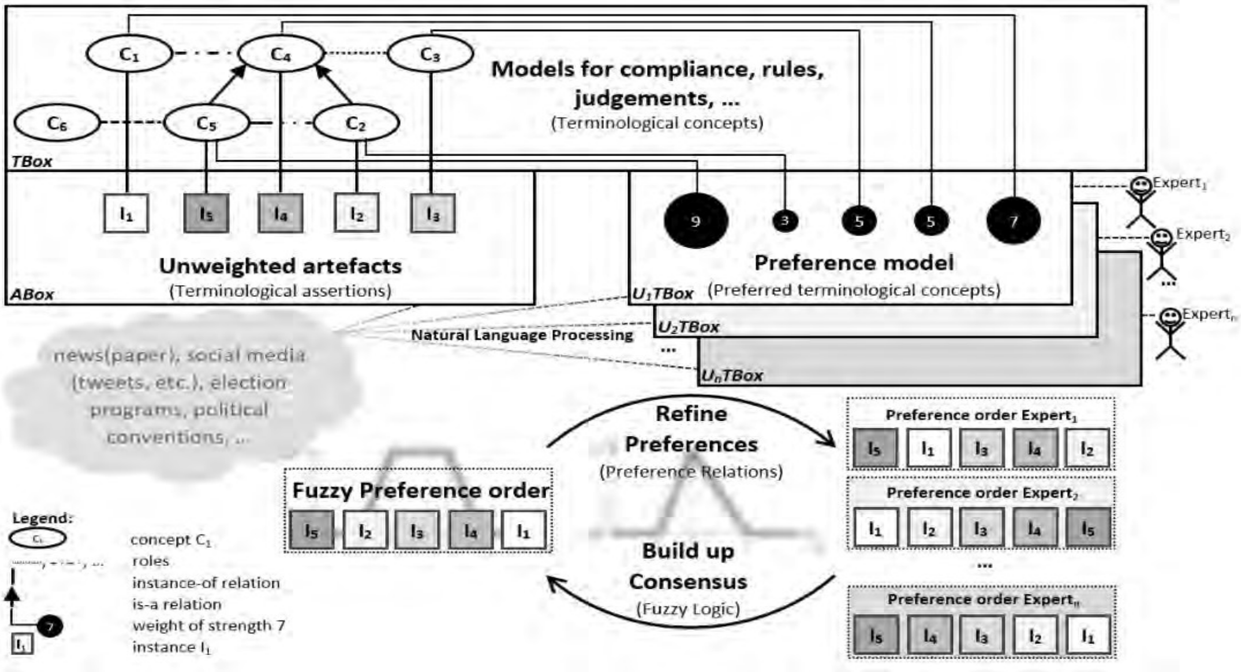
The original Opinion Mining Architecture (OMA) is part of a project of the same name. OMA was used for the first time for sentiment analysis from tweets for the financial sector [9]. To achieve an automated calculation of sentiment scores from texts, traditional approaches of natural language processing (such as POS tagging, parsing) and machine learning from texts (such as n-gram, syntactic/semantic features) were used for the preprocessing of the texts [10]. In addition, an extension of description logic [11], the so-called weighted description logic [6], was used to automatically calculate the sentiment scores. The idea of separating the text processing task (filtering out relevant phrases) and the decision support task (evaluating extracted phrases) comes from the text understanding system SYNDIKATE [12] and its qualitative calculus [4].

In order to explain the extension of OMA to include consensus mining and decision making, we first clarify the essential components of the OMA. In **Fig. 1** we see (from top to bottom):

- the $TBox$, which accommodates models via compliances, rules, judgements, etc.
- the $ABox$, which contains unweighted statements on the model of the $TBox$
- U_iBox (on the right side), that contain different preference models of experts

From a technical point of view, the models of the TBox are entirely expressed in description logic by means of terminological concepts, roles and is-a-relations. The elements of the ABox are terminological assertions that enter into an instance-of-relationship with concepts of the TBox. At this point it should be noted that these assertions will be created by the text-processing task from newspapers, social media, political programs, etc. (see the cloud on the left side in **Fig. 1**). The preference model of an expert e_i is shown in the U_iBox. A preference model consists of a priori preference relation over attributes of concepts (see black circles in **Fig. 1**). Each model represents the individual utility function of an expert e_i. With these a priori preference relations of an expert a first a posteriori preference order for each expert's choice can be derived (see the individual preference orders in **Fig. 1**). Note that the preference model of each expert can be extracted a priori by the text processing task or be entered directly by each expert. Next, the individual preference relations of each expert are used to build consensus or in case of only one expert to directly retrieve the best possible choice respective decision. The former is done by means of incomplete fuzzy preference relations for group decision making [13], which repeatedly adapts the preference relations of all experts until a satisfying consistent consensus is achieved. The theoretical basis of this approach comes from [14] and its IOWA operator. For more details see [15].

Fig. 1. The Opinion & Consensus Mining Architecture OMA



2.2 Description Logic

Description Logics (DLs) [11] are a family of logic-based knowledge representation formalisms. They can be used to represent and reason on the knowledge of an application domain. The basis of description logics is a common family of languages, known as description languages, which have a set of constructors to ontologies consisting of create concept (class) and role (property) descriptions.

A description language consists of an alphabet with unique concept names (N_C), role names (N_R) and individual (object) names (N_I). In addition, so-called constructors are used to create concept and role descriptions. Depending on which constructors are allowed, there are many different description languages. Some of them form the basis for the ontology language of the semantic web [16].

The $\mathcal{SROIQ}$ Description Logic. An expressive description language is called $\mathcal{SROIQ}$-$\mathcal{DL}$ [17]. $\mathcal{SROIQ}$-$\mathcal{DL}$ is compatible with OWL2, the current standard of the semantic web [18] and thus the most reasonable description language within the architecture introduced above. A formal definition of the notions $\mathcal{SROIQ}$-roles and $\mathcal{SROIQ}$-concepts, as well as the underlying model-theoretic semantics (the interpretation is written as I, the domain as Δ^I and the interpretation function as $\cdot^I$) can be found in [17]. Below are some examples of the syntax and semantics of $\mathcal{SROIQ}$-$\mathcal{DL}$:

Table 1.: Example syntax and semantics of $\mathcal{SROIQ}$-DL

Constructor	Syntax	Semantics
Top	$\top$	$\Delta^{\mathcal{I}}$
bottom	$\bot$	$\emptyset$
general negation	$\neg C$	$\Delta^{\mathcal{I}} \backslash C^{\mathcal{I}}$
conjunction / disjunction	$C \sqcap D$ / $C \sqcup D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}} / C^{\mathcal{I}} \cup D^{\mathcal{I}}$
exists restriction	$\exists R.C$	$\{x \in \Delta^{\mathcal{I}} \mid \exists y. \langle x, y \rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
value restriction	$\forall R.C$	$\{x \in \Delta^{\mathcal{I}} \mid \forall y. \langle x, y \rangle \in R^{\mathcal{I}} \rightarrow y \in C^{\mathcal{I}}\}$
at-most restriction	$\leq nR$	$\{x \in \Delta^{\mathcal{I}} \mid \#\{y \in \Delta^{\mathcal{I}} \mid R^{\mathcal{I}}(x, y)\} \leq n\}$
at-least restriction	$\geq nR$	$\{x \in \Delta^{\mathcal{I}} \mid \#\{y \in \Delta^{\mathcal{I}} \mid R^{\mathcal{I}}(x, y)\} \geq n\}$
concept definition / concept specialisation	$D \equiv C$ / $D \sqsubseteq C$	$D^{\mathcal{I}} = C^{\mathcal{I}} / D^{\mathcal{I}} \subseteq C^{\mathcal{I}}$

In DLs, we distinguish between terminological knowledge (so-called $\mathcal{T}$Box) and assertional knowledge (so-called $\mathcal{A}$Box). A $\mathcal{T}$Box is a set of concept inclusions $C \sqsubseteq D$ and concept definitions $C \equiv D$. An $\mathcal{A}$Box is a set of concept assertions $a{:}\,C$ as well as role assertions $(a, b){:}\,R$.

A so-called concrete domain $\mathcal{D}$ is defined as a pair $(\Delta^{\mathcal{D}}, pred(\mathcal{D}))$. $\Delta^{\mathcal{D}}$ is the domain of $\mathcal{D}$ and $pred(\mathcal{D})$ is the set of predicate names of $\mathcal{D}$. The following assumptions have been applied: $\Delta^{\mathcal{I}} \cap \Delta^{\mathcal{D}} = \emptyset$ and for each $P \in pred(\mathcal{D})$ with arity n there is $P^{\mathcal{D}} \subseteq (\Delta^{\mathcal{D}})^n$. According to [11], functional roles are denoted with lower case letters, for example with r. In description logics with concrete precise domains, N_R consists of functional roles and ordinary roles. A role r is functional if for every $(x, y) \in r$ and $(w, z) \in r$ it is necessary that $x = w \Rightarrow y = z$. Functional roles are explained as partial functions from $\Delta^{\mathcal{I}}$ to $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{D}}$. Within $\mathcal{SROIQ}$ all statements gathered about roles are captured in an $\mathcal{R}$Box, which for the sake of convenience and for compatibility to the definitions in [11] is not applied to our examples.

Next, we will build a knowledge base (originally introduced in [9]) of a domain that will be used in the further course of the work. Its purpose is pure illustrative, so that reasoning and entailment is obvious. We will note explicit and implicit knowledge (*"- ik -"*):

$\mathcal{T} = \{Device \sqsubseteq \top, Equip \sqsubseteq \top, Device \sqcap Equip \sqsubseteq \bot, PoorEquip \sqsubseteq Equip, WellEquip \sqsubseteq Equip,$
$PoorEquip \sqcap WellEquip \sqsubseteq \bot, Device \equiv \exists hasWeight.>_{0g} \sqcap \exists hasPrice.>_{0€} \sqcap \forall equipped.Equip,$
$Tablet \equiv Device \sqcap \exists hasPrice.>_{200€}, InexpensiveTablet \equiv Tablet \sqcap \exists hasPrice.\leq_{500€},$
$ExpensiveTablet \equiv Tablet \sqcap \exists hasPrice.\geq_{900€}, InexpensiveTablet \equiv \neg ExpensiveTablet,$
$ExpensiveTablet \equiv \neg InexpensiveTablet, LightWeightTablet \equiv Tablet \sqcap \exists hasWeight.\leq_{900\,g},$
$Convertable \sqsubseteq UpperclassTablet, UpperclassTablet \equiv Tablet \sqcap \forall equipped.WellEquip,$
$LowerclassTablet \equiv Tablet \sqcap \forall equipped.PoorEquip,$
$UpperclassTablet \sqcap LowerclassTablet \sqsubseteq \bot,$ *- ik -*}

$\mathcal{A} = \{tab_1{:}\,Tablet, (tab_1, 999€){:}\,hasPrice, (tab_1, 710g){:}\,hasWeight,$
$equipment_1{:}\,WellEquip, (tab_1, equipment_1){:}\,\forall equipped.WellEquip,$
$tab_1{:}\,ExpensiveTablet, tab_1{:}\,LightWeightTablet,$ *- ik -*
$tab_1{:}\,UpperclassTablet,$ *- ik -*
$tab_2{:}\,Tablet, (tab_2, 399€){:}\,hasPrice, (tab_2, 1250g){:}\,hasWeight, equipment_2{:}\,PoorEquip,$
$(tab_2, equipment_2){:}$
$\forall equipped.PoorEquip, tab_3{:}\,Tablet, (tab_3, 600€){:}\,hasPrice, tab_3{:}\,Convertable,$
$tab_2{:}\,InexpensiveTablet, tab_2{:}\,LowerclassTablet,$ *- ik -*
$tab_3{:}\,UpperclassTablet, equipment_3{:}\,WellEquip,$ *- ik -*
$(tab_3, equipment_3){:}\,\forall equipped.WellEquip$ *- ik -*}

A considerable amount of knowledge is implicitly revealed in the terminological knowledge base. In this case, a lot of knowledge is available about the domain but no support to make a comprehensible decision. For this reason, the capabilities of the knowledge base are extended by the possibility to depict individual preferences.

2.3 Weighted Description Logic

Weighted description logic (WDL) can be regarded as a generic framework, the so-called decision base [6]. We use an a priori preference relation over attributes (called the ontological classes). Thereby, an *a posteriori* preference relation over choices (called *ontological individuals*) can be derived. Formally, a utility function U over $\mathcal{X}$ (the set of attributes) is defined ($U: \mathcal{X} \rightarrow \mathbb{R}$). Additionally, a utility function u defined over choices, which uses logical entailment, extends the utility function U to the subset of choices [19]. Modelling attributes takes place in two steps:

1. Each attribute is modelled by a concept
2. For every value of an attribute a new (sub)concept is introduced

For instance, if *equipped* is an attribute to be modelled, it is simply represented by the concept $Equipment$ (i.e. $Equipment \in \mathcal{X}$). An equipment can be regarded as a value, as if it was a concept of its own. If "*well equipped"* is a value of the attribute *equipped*, the attribute set $\mathcal{X}$ is simply extended by adding the concept $WellEquip$, as a sub-concept of $Equipment$. It should be noted, that an axiom is introduced to guarantee the disjointedness (e.g. $PoorEquip \sqsubseteq \neg WellEquip$) and that this procedure results in a binary term vector for $\mathcal{X}$, because an individual c (as a choice) is either a member of a specific attribute of the concept set $\mathcal{X}$ or not.

Given a total preference relation (i.e. $\succcurlyeq_{\mathcal{X}}$) over an ordered set of not necessarily atomic attributes $\mathcal{X}$, and a function $U: \mathcal{X} \rightarrow \mathbb{R}$ that represents $\succcurlyeq$ (i.e., $U(X_1) \geq U(X_2)$ **iff** $X_1 \succcurlyeq_{\mathcal{X}} X_2$ for $X_1, X_2 \in \mathcal{X}$). The function U assigns an *a priori* weight to each concept $X \in \mathcal{X}$. Therefore, one can say, that "U makes the description logic weighted". The utility of a concept $X \in \mathcal{X}$ is denoted by $U(X)$. The following applies: The greater the utility of an attribute the more the attribute is preferable.

As mentioned above, a *choice* is an individual $c \in N_I$. $\mathcal{C}$ denotes the finite set of choices. To determine a preference relation (*a posteriori*) over $\mathcal{C}$ (i.e. $\succcurlyeq_{\mathcal{C}}$), which respects $\succcurlyeq_{\mathcal{X}}$, a utility function $u(c) \in \mathbb{R}$ is introduced. $u(c)$ indicates *the utility of a choice* c relative to the attribute set $\mathcal{X}$. Also, a utility function U over attributes as an aggregator is introduced. For simplicity, the symbol $\succcurlyeq$ is used for both choices and attributes whenever it is evident from the context.

Within a consistent knowledge base $\mathcal{K} \coloneqq \langle \mathcal{T}, \mathcal{A} \rangle$, consisting of a $\mathcal{T}$Box $\mathcal{T}$ and an $\mathcal{A}$Box $\mathcal{A}$, the σ-utility is a particular u and is defined as $u_\sigma(c) \coloneqq \sum\{U(X) \mid X \in \mathcal{X}$ and $\mathcal{K} \models \mathrm{c}: X\}$ and is called the *sigma utility of a choice* $c \in \mathcal{C}$. u_σ triggers a preference relation over $\mathcal{C}$ i.e., $u_\sigma(c_1) \geq u_\sigma(c_2)$ **iff** $c_1 \succcurlyeq c_2$. Each choice corresponds to a set of attributes, which is logically *entailed* e.g. $\mathcal{K} \models \mathrm{c}: X$. Due to the criterion additivity, each selection c corresponds to a result.

Putting things (DL, U and u) together, a generic $\mathcal{U}$Box (so-called *Utility Box*) is defined as a pair $\mathcal{U} \coloneqq (u_\sigma, U)$, where U is a utility function over $\mathcal{X}$ and u_σ is the utility function over $\mathcal{C}$. Also, a decision base is defined as a triple $D = (\mathcal{K}, \mathcal{C}, \mathcal{U})$ where $\mathcal{C} \subseteq N_I$ is the set of choices and $\mathcal{U} = (u, U)$ is an $\mathcal{U}$Box. Note: $\mathcal{K}$ provides assertional information about the choices and terminological information about the agent ability to reason over choices.

Now, we expand our tablet example by using different utility boxes ($\mathcal{U}_i$) resp. utility functions ($u_{i,\sigma}$) of two experts:

- For expert 1 applies $\mathcal{U}_1 = \{(InexpensiveTablet, 50), (UpperClassTablet, 40), (LightWeightTablet, 40)\}$, $u_{1,\sigma}(tab_1) = 40 + 40 = 80$, $u_{1,\sigma}(tab_2) = 50$ and $u_{1,\sigma}(tab_3) = 40$. It follows that $tab_1 \succ tab_2 \succ tab_3$.
- For the expert 2, however, applies $\mathcal{U}_2 = \{(InexpensiveTablet, 60), (UpperClassTablet, 20), (LightWeightTablet, 10)\}$, $u_{2,\sigma}(tab_1) = 20 + 10 = 30$, $u_{2,\sigma}(tab_2) = 60$ and $u_{2,\sigma}(tab_3) = 20$. It follows, that $tab_2 \succ tab_1 \succ tab_3$.

Within this decision base an expert with the utility box $\mathcal{U}_1$ would classify tab_1 as first choice whereas an expert with a different utility box in this example $\mathcal{U}_2$ would prefer tab_2. Two different problems appear looking at tab_3. One is that for this tablet a weight (in the sense of mass not weighting of a concept according to WDL sense) is not known. Therefore, the reasoning fails doing an instance check for this tablet on the concept *LightWeightTablet*. For this reason, when calculating the utility value, it is treated as it would not be an instance of *LightWeightTablet*. But the membership of this concept is unknown, and one cannot reason that the instance does not belong to the concept *LightWeightTablet*. The other problem is that at a price of 600€, the tablet is neither inexpensive nor expensive (*InexpensiveTablet* resp. *ExpensiveTablet)*. Although the price is well-known, the utility function treats this tablet the same way as expensive ones, which is not quite reasonable in this scenario. To eliminate this problem the knowledge base is extended by fuzzy description logic and then combined with the decision base, which is introduced in subsequent chapters.

2.4 Fuzzy Description Logic

To deal with the ambiguity of the underlying domain, it is necessary to clarify, where this uncertainty comes from. Either the uncertainty is due to a probabilistic cause or due to vagueness. If the first situation occurs, then a statement is either "true" or "false" to some possibility (in the sense of likelihood), whereas in the second situation, a statement is to some degree (in the sense of reaching a graded level) either "true" or "false". For more information on these two approaches, see [20].

In the context of the above choice of tablets, the underlying ambiguity arises from vagueness. To model vague knowledge description logic is enriched with fuzzy logic, which enables reflecting the degree of membership to a certain concept. According to [21] a fuzzy set is defined by its characteristics the so-called membership function.

Let X be a non-empty set of individuals, then a class A in X is characterized by its membership function $f_A: X \to [0,1]$ and assigns to each $x \in X$ a real number within the interval [0,1]. This value represents how large the degree of its membership to A is. The membership function defined for fuzzy sets fulfills some essential properties which appear to be natural, see also [22]:

- $\forall x \in X: A = \emptyset$ **iff** $f_A(x) = 0$
- $\forall x \in X: A' = X \setminus A: f_{A'}(x) = 1 - f_A(x)$
- $\forall x \in X: A \subseteq B: f_A(x) \leq f_B(x)$
- $\forall x \in X: A \cup B: f_{A \cup B}(x) = \max(f_A(x), f_B(x))$
- $\forall x \in X: A \cap B: f_{A \cap B}(x) = \min(f_A(x), f_B(x))$

To augment the possibilities of fuzzy sets, algebraic operations can also be defined. There are plenty of definitions e.g. Łukasiewicz logic, Gödel logic. For a detailed comprehension of these algebraic operations and their relation to DLs, see [23]. In this work, the standard fuzzy logic (SFL) is used, but all others can be applied as well. Some definitions can be found in table 2.

This toolset of fuzzy set definitions and algebraic operators can now be applied to description logics to reflect ambiguity and vagueness in knowledge bases. Hence, an individual that is an instance of a concept only to a certain degree for example can be modelled suitably.

Table 2. Definitions of algebraic operations

Algebraic operator	SFL
$a \otimes b$ / $a \oplus b$	$\min(a,b)$ / $\max(a,b)$
$a \Rightarrow b$ / $\ominus a$	$\max(1-a,b)$ / $1-a$

To formally quote this fuzziness of description logic axioms we use the syntax of [7]. The conceptional syntax of fuzzy description logics is the same as for description logics defined above (see chapter 2.2). The semantic however reflects the fuzzy logic. Therefore, a *fuzzy interpretation* is a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consisting of a non-empty set called the domain and a fuzzy interpretation function. This function maps individuals as usual and concepts into membership functions $\Delta^{\mathcal{I}} \rightarrow [0,1]$. Accordingly, the roles are mapped into $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \rightarrow [0,1]$. Consequently $C^{\mathcal{I}}$ is the membership function of the fuzzy set C. Hence, a concept is interpreted as fuzzy set.

Example

A specific *tablet* is an instance of the concept *Convertable* only to a certain degree depending on its features. We therefore extend the description logic and allow capturing this degree as a fuzzy value and write $\langle tab_3\colon Convertable, 0.8\rangle$. This means that tab_3 is at least an instance of the concept $Convertable$ with the degree of 0.8. Analogously, $Convertable^{\mathcal{I}}(tab_3)$ gives back the minimal degree of tab_3 being a convertable tablet under the interpretation $\mathcal{I}$.

The properties of fuzzy sets and algebraic operators are now applied to interpretations of $\mathcal{SROIQ}$ and, according to [24], lead to the following example rules for all $d \in \Delta^{\mathcal{I}}$ (non-exhaustive list):

Table 3. Fuzzy semantics

Syntax	Semantics
$C \sqcap D$	$(C \sqcap D)^{\mathcal{I}}(d) = \min\{C^{\mathcal{I}}(d), D^{\mathcal{I}}(d)\}$
$\neg C$	$(\neg C)^{\mathcal{I}}(d) = 1 - C^{\mathcal{I}}(d)$
$C \sqsubseteq D$	$(C \sqsubseteq D)^{\mathcal{I}} = \inf_{d\in\Delta^{\mathcal{I}}} C^{\mathcal{I}}(d) \Rightarrow D^{\mathcal{I}}(d)$
$\exists R.C$	$(\exists R.C)^{\mathcal{I}}(a) = \sup_{b\in\Delta^{\mathcal{I}}}\{\min(R^{\mathcal{I}}(a,b), C^{\mathcal{I}}(b))\}$

Example

Let $\mathcal{K}$ be the knowledge base above, but now the concept $LightWeightTablet$ is not strictly defined according to classic DL, but intuitively with the help of fuzzy DL. Within the $\mathcal{T}$Box, the row $\exists hasWeight.\leq_{900g} \sqsubseteq LightWeightTablet$ will be replaced by the following two constructs:

$$\langle(\exists hasWeight._{\geq 900g} \sqcap \exists hasWeight._{\leq 1100g}) \sqsubseteq LightWeightTablet, 0.6\rangle$$
$$\langle \exists hasWeight._{\leq 900g} \sqsubseteq LightWeightTablet, 1\rangle$$

indicating that every tablet which has a weight less than 1100g should still be considered as a light tablet to a certain degree (here 0.6). For tab_3 the exact weight is not known but relating information vary between 900g an 1100g with strong tendencies to the upper threshold. Therefore, the $\mathcal{A}$Box is adjusted accordingly: $\langle tab_3\colon \exists hasWeight._{\geq 900g}, 0.5\rangle$ and $\langle tab_3\colon \exists hasWeight._{\leq 1100g}, 0.9\rangle$. The $\mathcal{T}$Box reveals $tab_3\colon \exists hasWeight._{\geq 900g} \sqcap tab_3\colon \exists hasWeight._{\leq 1100g}$

$$= \min\{tab_3\colon \exists hasWeight._{\geq 90}\quad, tab_3\colon \exists hasWeight._{\leq 1100g}\} = \min\{0.5, 0.9\} = 0.5$$

and tab_3 is therefore a $LightWeightTablet$ with the minimal degree of $\max\{1 - 0.5, 0.6\} = 0.6$.

3 Weighted Fuzzy Description logic

For the weighted fuzzy description logic, the background knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ will be allowed to capture also vague knowledge and assertions, which is formally noted as $\mathcal{K} \approx (\mathcal{T}, \mathcal{A})$. This knowledge base is then extended by the set of choices $\mathcal{C}$ and the utility box $\mathcal{U}$ steering the decision making. The advantage of this framework is, that these weights can be independently articulated and do not need to be compared against each other like in [25].

Definition
A triple $\mathcal{D} \approx (\mathcal{K}, \mathcal{C}, \mathcal{U})$, where $\mathcal{K}$ is a fuzzy knowledge base, $\mathcal{C}$ a set of choices and $\mathcal{U}$ a utility box is called a *fuzzy decision base*.

Note: Entities of the $\mathcal{U}$Boxes are concepts relevant to the decision-making process, including their specific individual weights. After the reasoning for each of the existing choices, the instance check completes and reveals whether a choice belongs to a specific concept or not. Assume a choice $c: U$ belongs to a concept, then this might also be vague within the fuzzy description logics. Hence it leads to constructs like $\langle c: U, n\rangle$ with $n \in [0,1]$.

Definition
Let $\langle c: U, n\rangle$ be a fuzzy assertion and (U, w) a weighted attribute, then a *fuzzy utility value of c respective to U* is $u_{f\sim\sigma}(c: U) \stackrel{\text{def}}{=} w \cdot n$

If the assertion is not fuzzy, then n is simply set to 1. If the instance c belongs to the complement of U with a membership degree of 1, then the fuzzy utility value for c on this attribute is 0 (as n is then 0).

Example
In case for tab_3 the calculated respective reasoned membership degree to a light weight tablet is 0.6, formally written $\langle tab_3: LightWeightTable, 0.6\rangle$ and expert U_1 defines a weight of 40 for this attribute, formally written as $(LightWeightTablet, 40)$, then $u_{f\sim\sigma}(tab_3) = 24$. The individual weight is a bit more than half of the initially defined one, as the degree of membership for this tablet is only 0.6. As the utility function is additive the utility measure for a choice is the sum across all relevant attributes.

Definition
Let $\mathcal{D} \approx (\mathcal{K}, \mathcal{C}, \mathcal{U})$ be a fuzzy decision base with a utility box $\mathcal{U}$ of the cardinality $k = |\mathcal{U}|$ then the *$\mathcal{U}$Box fuzzy utility value of c* is $u_{f\sim\mathcal{U},\sigma} \stackrel{\text{def}}{=} \sum_{i=1}^{k} u_{f\sim\sigma}(c: U_i)$.

By calculating the $\mathcal{U}$Box fuzzy utility values of each choice *c*, a total ordering of the set $\mathcal{C}$ is naturally given. The ideal solution is therefore the choice with the highest fuzzy utility value relative to the $\mathcal{U}$Box.

Definition
Let $\mathcal{D} \approx (\mathcal{K}, \mathcal{C}, \mathcal{U})$ be a fuzzy decision base with a utility box $\mathcal{U}$ of the cardinality $k = |\mathcal{U}|$ then *the ideal fuzzy choice* is $c_{f\sim s} \stackrel{\text{def}}{=} \arg\max_{c \in \mathcal{C}}(\sum_{i=1}^{k} u_{f\sim\sigma}(c: U_i))$.

Example
For an expert with the utility box U_1 the summarized utility value for tab_3 is $\sum_{i=1}^{k} u_{f\sim\sigma}(c: U_i) = 24 + 40 = 64$. The ranking of choices for this expert, changes to $tab_1 \succ tab_3 \succ tab_2$, which means that $ta\ _3$ is preferred to tab_2. In this scenario, the problem remains that the price of the tablet is neither expensive nor inexpensive, but unknown. Therefore, it is indispensable to design a consistent respective complete fuzzy decision base by ensuring that each attribute listed in the $\mathcal{U}$Box is correct and decidable in the knowledge base.

Definition
A fuzzy decision base $\mathcal{D} \approx (\mathcal{K}, \mathcal{C}, \mathcal{U})$ is called *complete*, if for every relevant attribute out of the $\mathcal{U}$ Box $U \in \mathcal{U}$ a fuzzy value for every $c \in \mathcal{C}$ is deducible: $\forall\, U \in \mathcal{U}, \forall c \in \mathcal{C}\ \exists n \in [0,1]: \langle c: U, n \rangle$. Thus, the fundamentals are defined to make a reasonable decision in the above scenario.

4 Results and Outlook

To complete the fuzzy decision base of the example above, tab_3 requires a fuzzy value for the attribute "$InexpensiveTable$ " and tab_2 for "$LightWeightTablet$". Therefore the $\mathcal{T}$Box is extended to reveal fuzzy values also for weights above 1100g and prices in between 500€ and 900€. The following expressions are added to the $\mathcal{T}$Box:

$\langle \neg\exists hasWeight._{\leq 1100g} \sqsubseteq LightWeightTablet, 0 \rangle$
$\langle (\exists hasPrice._{>500€} \sqcap \exists hasPrice._{<900€}) \sqsubseteq InexpensiveTablet, 0.5 \rangle$
$\langle (\exists hasPrice._{>500€} \sqcap \exists hasPrice._{<900€}) \sqsubseteq ExpensiveTablet, 0.5 \rangle$

The first line indicates that tablets with a weight above 1100g do not belong to the concept $LightWeightTablet$ at all. The fuzzy values of the second and third line represent the membership degrees of those tablets which have a price within this interval and is set manually to 0.5 as arithmetic mean between the two categorizations inexpensive and expensive. Thus, tab_3 is a member of the concept *Inexpensive* with a degree of 0.5 and a member of *Expensive* with the same degree.

With standard fuzzy logic the fuzzy value of a concept's complement is: $(\neg C)^{\mathcal{I}}(d) = 1 - C^{\mathcal{I}}(d)$, which entails the following implicit knowledge:

$$\langle InexpensiveTablet \sqcap \exists hasPrice._{\geq 900€}, 0 \rangle$$

For example, if tab_1 has the price of 999€, then the following applies:

$$\langle tab_1: ExpensiveTablet, 1 \rangle \text{ and } \langle tab_1: \neg ExpensiveTablet, 0 \rangle$$

By means of this complete decision base the above decision can be derived properly. The utility values for tab_3 within $\mathcal{U}_1$ and $\mathcal{U}_2$ are:

$$u_{f\sim 1,\sigma}(tab_3) = 0.5 \cdot 50 + 40 + 0.6 \cdot 40 = 89$$
$$u_{f\sim 2,\sigma}(tab_3) = 0.5 \cdot 60 + 20 + 0.6 \cdot 10 = 56$$

For an expert with the utility box U_1 tab_3 is his or her first choice, while the other expert still chooses tab_2. Through the augmented decision base by fuzzy logic a model is defined, which represents reality much better. Because there was uncertainty around tab_3 a first calculation in a conventional decision base revealed a distorted result. By incorporating the vague knowledge existing in this domain, the expert would have chosen tab_3 instead of tab_1.The strength of this framework is that vague assertions together with individual preferences are deliberated properly.

Also, it becomes obvious how weighting influences the decision. As the first utility box has almost balanced weights, the second one has a strong tendency towards inexpensive tablets. Using this $\mathcal{U}$Box the first choice is still tab_2. But the second choice is now tab_3 and not tab_1. Both tablets were initially not belonging to the concept $InexpensiveTablet$, but with the help of fuzzy logic, the strong preference for inexpensive tablets causes that tab_3 passes tab_1.

Summarized, complete fuzzy decision bases offer a strong possibility to model real world situations which need to respect ambiguity and individual preferences and at the same time support a comprehensible decision-making process. Further researches need to reveal supporting algorithms to detect and locate incompleteness to support the creation of complete fuzzy decision bases. Overall, the creation of these underlying ontologies is time-consuming and a non-trivial, manual process. To facilitate this, new approaches with deep learning algorithms have risen [26].

The use of such techniques is another milestone on the way to a fully automated decision making process.

References

1. Jung, J., Lee, H., Choi, K.: Contextualized Recommendation Based on Reality Mining from Mobile Subscribers. Cybernetics and Systems. 40(2), 160-175 (2009)
2. Gigerenzer , G., Gaissmaier, W.: Heuristic Decision Making. Annual Review of Psychology 62, 451-482 (2011)
3. Keeney, R., Raiffa, H.: Decisions with Multiple Objectives: Preferences and Value Trade-offs. Cambridge University Press, First published in 1976 by John Wiley & Sons, Inc. (1993)
4. Schnattinger, K., Hahn, U.: Quality-Based Learning. ECAI'98: Proc. 13th Biennial European Conference on Artificial Intelligence, Brighton, UK,160-164 (1998)
5. Lafage, C., Lang, J.: Logical Representation of Preferences for Group Decision Making. KR'00: Proc. 7th Conference on Principles of Knowledge Representation and Reasoning, 457-468 (2000)
6. Acar, E., Fink, M., Meilicke, C., Thome, C., Stuckenschmidt, H.: Multi-attribute Decision Making with Weighted Description Logics. IFCoLog: Journal of Logics and its Applications 4, 1973-1995 (2017)
7. Straccia, U.: Reasoning within Fuzzy Description Logics. Journal of Artificial Intelligence Research(14), 137-166 (2001)
8. Schnattinger, K., Mueller, N., Walterscheid, H.: Consensus Mining - A Guided Group Decision Process for the German Coalition Negotiations. FLAIRS'18: Proc. 31st Florida Artificial Intelligence Research Symposium, Melbourne, USA, 205-208 (2018)
9. Schnattinger, K., Walterscheid, H.: Opinion Mining Meets Decision Making: Towards Opinion Engineering. IC3K'17: Proc. 9th International Joint Conference on Knowledge Discovery, Knowledge Engineering and Knowledge Management, 334-341 (2017)
10. Sun, S., Luo, C., Chen, J.: A Review of Natural Language Processing Techniques for Opinion Mining Systems. Information Fusion 36, 10-25 (2017)
11. Baader, F., McGuinness, D., Narci, D., Patel-Schneider, P.: The Description Logic Handbook: Theory, Imlementation, and Applications. Cambridge University Press (2003)
12. Hahn, U., Schnattinger, K.: Towards Text Knowledge Engineering. AAAI'98: Proc. 15th National Conference on Artificial Intelligence, 524-531 (1998)
13. Herrera-Viedma, E., Alonso, S., Chiclana, F., Herrera, F.: A Consensus Model for Group Decision Making Incomplete Fuzzy Preference Relations. IEEE Transactions on Fuzzy Sytems 15(5), 863-877 (2007)
14. Yager, R. R., Filev, D. P.: Operations for Granular Computing: Mixing Words and Numbers. IEEE International Conference on Fuzzy Systems 2(1), 123-128 (1998)
15. Mueller, N., Schnattinger, K., Walterscheid, H.: Combining Weighted Description Logic with Fuzzy Logic. IPMU'18: Proc. 17th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, 124-136 (2018)
16. Horrocks, I., Patel-Schneider , P. F., McGuinness, D. L., Welty, C. A.: OWL: a Description Logic Based Ontology Language for the Semantic Web. In : Description Logic Handbook, 458-486. Cambridge University Press, Cambridge (2007)
17. Horrocks, I., Sattler, U., Kutz, O.: The even more irresistible SROIQ. KR'06: Proc. 10th Conference on Principles of Knowledge Representation and Reasoning, 57-67 (2006)

18. Motik, B., Patel-Schneider, P. F., Cuenca Grau, B.: OWL 2 Web Ontology Language: Direct Semantics (Second Edition). (Accessed April 2018) Available at: http://www.w3.org/TR/2012/REC-owl2-direct-semantics-20121211/
19. Fishburn, P.: Utility Theory for Decision Making. R. E. Krieger Publications & Co, Huntington, N.Y (1979)
20. Dubois, D., Prade, H.: Possibility theory, probability theory and multiple-valued. Annals of Mathematics and Artificial Intelligence 32(1-4), 35-66 (2001)
21. Zadeh, L. A.: A Computational Approach to Fuzzy Quantifiers in Natural Languages. Computers & Mathematics with Applications 9(1), 149-184 (1983)
22. Hájek, P.: Metamathematics of Fuzzy Logic. Springer, Dordrecht; Netherlans (1998)
23. Straccia, U.: All About Fuzzy Description Logics and Applications. In Faber, W., Paschke, A., eds. : Reasoning Web. Web Logic Rules, Cham, 1-31 (2015)
24. Stoilos, G., *et.al.*: Reasoning with Very Expressive Fuzzy Description Logics. Journal of Artificial Intelligence Research (30), 273–320 (2007)
25. Yager, R., Basson, D.: Decision Making with Fuzzy Sets. Fuzzy Sets and Systems, 87-95 (1978)
26. Petrucci G., Ghidini C., Rospocher M.: Ontology Learning in the Deep. In: Blomqvist E., Ciancarini P., Poggi F., Vitali F. (eds) Knowledge Engineering and Knowledge Management. EKAW 2016. Lecture Notes in Computer Science, Springer, 480-495 (2016)